Uncertainty and a 7-Valued Logic

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***Abstract*— Uncertainty is a research topic which has not only been well known to the AI community but also, more recently, in the programming languages community at some extent. This paper joins the well known uncertainty model of MYCIN, extended, with a seven-valued deductive logic introduced here, working towards the unification of two different forms of representation, and of reasons, as humans always do. This seven-valued logic is introduced by myself as a novelty, here in the present paper, after having proposed a similar but simplified five-valued logic and a system, but this seven-valued logic is fully compatible with the same uncertainty model.**

I. Introduction

Uncertainty and fuzziness are research topics which are well known to the artificial intelligence and programming languages communities[1], and some commercial expert systems environments even provide models for treating uncertainty, most commonly, they are based on production systems with confidence factors. In the last two decades, most expert systems have been based on some uncertainty model. On the other hand, it is also well known that fuzzy systems[3] have been successfully used in industry. I would say that deduction and logics are *analytical* whereas fuzzy logics and uncertainty simulates the *synthesis*. Further, these two pairs of concepts are complementary forms of reasoning and they ought to work together. The notion of uncertainty itself implies uncertainty in its use. Therefore, there cannot be a universal proof that a model of uncertainty is more realistic than another one, although there is common sense and sometimes consensus or diverging opinions. It is important to say that the lack of proof over subjective issues is not a problem either, but instead part of the reality, which is very subjective in the most general meaning of reality. In contrast, there can also be statements such as "clearly, the model X is more realistic than the model Y" while one rely upon the agreement from the others. The MYCIN system[2] has the reputation of the University of Stanford and, in my opinion, its model of uncertainty sounds natural. In this way, uncertainty models that require many probabilities are mathematically accurate, and they may be the most appropriate in some cases. However, e.g. for agents, probabilities may not be the most suitable model for representing knowledge or belief, in comparison to vaguer and less precise notions[4]. I refer to the latter form of inference as uncertainty-based inference.

On the other hand, the present work makes use of a weaker and simplified semantics for inconsistency, in comparison to da Costa's paraconsistent logics[5]. While his diagrams are in 2D, my diagrams are linear because inconsistency is not taken as orthogonal to uncertainty.

In addition to uncertainty, in a context such as global computing[6], when connections fail or delay, programs should carry on running despite the lack of information. uu, which means "unknown" or "undefined", is a constant in programming languages that can be assigned to any variable of any data type. This new constant guarantees both safety and robustness at the same time, because variables are never committed to any value that is not in the problem domain, while uu is not in the problem domain. A specific discussion on uu is in [7].

In this paper, I present an uncertainty model which is part of the Plain programming language, developed by myself, and introduce as a novelty a seven-valued logic fully compatible with this model. [1] adapts the MYCIN model and brings the adapted model to the language level, in particular for global computing, and provides a formalization to the model. Thus, the present paper continues the former by introducing a seven-valued logic.

The other sections are organized as follows: Section II presents related concepts and implicitly shows that there exists concept of truth which is subjective here. Section III introduces the seven-valued logic, and section IV contains the conclusion.

II. *uu* AND UNCERTAINTY

When an agent contains a great number of hypotheses, the system is deemed to be intelligent, with the ability of making decisions given the complex and subjective nature of the objects.

Initially, all hypotheses have value uu. Like measure, certainty factors are in the real interval $[-1.0, +1.0]$ and measure how the corresponding premises contribute to prove or refute the hypotheses, depending on the sign, positive or negative. Certainty factor is a part of a specification and certainty measure is a dynamic value in the same real interval $[-1.0, +1.0]$. In this way, each relation between one premise and one hypothesis has one certainty factor and one certainty measure attached. During the evaluation, a certainty measure is multiplied by the corresponding certainty factor, and its result is finally an input to the hypothesis.

By combining hypotheses and premises, one can represent complex knowledge forming an acyclic graph.

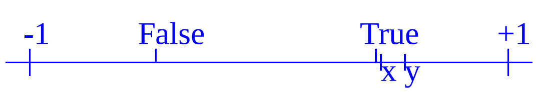
Within $[-1.0, +1.0]$, there are four subvariables: two thresholds, namely $False$ and $True$, where $False \leq True$, and minimum and maximum certainty measures, namely x and y, where $x \leq y$.

The next six figures below represent the common possible states of a logical variable. A possible initial state in which a variable can be regarded as uu if the condition $x < False \wedge y \geq True$ holds is as in the following diagram:

uu (White)

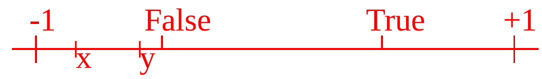
The above state is called uu. In a programming language system, the initial state is not necessarily $x = -1 \wedge y = +1$, but instead both values can be calculated by the compiler.

A variable can eventually be regarded as $true$ (the logical value is tt), if the condition $x \geq True$ holds:



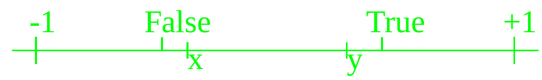
tt (Blue)

Or eventually, a variable can be regarded as $false$ (the logical value is ff), if the condition $y < False$ holds:



ff (Red)

Or eventually, a variable can be regarded as inconsistent (the value is ii), if the condition $x \geq False \wedge y < True$ holds:

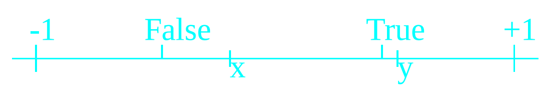


ii (Green)

There are other states such as *non-False* ($x \geq False \wedge x < True \wedge y \geq True$) and *non-True* ($x < False \wedge y \geq False \wedge y < True$). However, both are regarded as $unknown$ (uu) in [1], and inconsistency is interpreted in a different way.

Taking a particular hypothesis, the difference $y - x$ works in a particular experiment and means the amount of non-evaluated evidence in that experiment, while the difference $True - False$ means the amount of uncertainty upon past experience with respect to the hypothesis.

Now, I extend that mentioned uncertainty model [1] in the following way. As already suggested, from the uu state, it is possible that the variable enters in the state *non-False* (the value is it) by increasing the value of x:

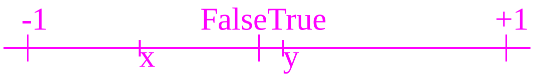


it (Cyan)

Accordingly, it is also possible from the uu state that the variable enters in the state *non-True* (the value is fi) by decreasing the value of y:

fi (Yellow)

Finally, since the experiment in question can be in parallel to others, from the state uu, the threshold $False$ can increase, or the threshold $True$ can decrease, in such a way that $False = True$ holds and in such a way that the state of inconsistency no longer exists with respect to the variable, as follows:



kk (Magenta)

For keeping the monotonicity in a sense, there are some constraints: $False$ and $True$ can increase or decrease if and only if both $False$ and $True$ keep the same relative positions with respect to x and y, except that $False = x$ leads to the same as $False < x$ does, and $True = y$ leads to the same as $True < y$ does.

I refer to the last state, where $x < False = True \leq y$, as kk, and that means that the value either $False$ or $True$ is (possibly) known and they are certainly consistent. Now there are seven states altogether. There can be three-valued systems that start having the kk value and work on the $\{ff, kk, tt\}$ values.

III. A Seven-Valued Logic

While working with the above model of uncertainty, I observed that it forms a seven-valued logics. However, so far, I have not seen any seven-valued logic in the literature.

Conjunction is represented by the following table:

$\wedge$	uu	kk	fi	ff	ii	tt	it
uu	uu	uu	fi	ff	fi	uu	uu
kk	uu	kk	fi	ff	fi	kk	uu
fi	fi	fi	fi	ff	fi	fi	fi
ff	ff	ff	ff	ff	ff	ff	ff
ii	fi	fi	fi	ff	ii	ii	ii
tt	uu	kk	fi	ff	ii	tt	it
it	uu	uu	fi	ff	ii	it	it

Disjunction is represented by the following table:

$\vee$	uu	kk	fi	ff	ii	tt	it
uu	uu	uu	uu	uu	it	tt	it
kk	uu	kk	uu	kk	it	tt	it
fi	uu	uu	fi	fi	ii	tt	it
ff	uu	kk	fi	ff	ii	tt	it
ii	it	it	ii	ii	ii	tt	it
tt	tt	tt	tt	tt	tt	tt	tt
it	it	it	it	it	it	tt	it

Both conjunction and disjunction are commutative connectives. Associativity is another property of both, and they have neutral elements that are tt and ff, respectively. Idempotency also holds for both connectives. Moreover, while the stronger equivalence is defined as $A \leftrightarrow B \overset{\text{def}}{=} A = B$,

$$A \wedge (B \vee C) \Leftrightarrow A \wedge B \vee A \wedge C$$

and

$$A \vee (B \wedge C) \Leftrightarrow (A \vee B) \wedge (A \vee C)$$

hold with my weaker equivalence connective, whose table is the following:

$\Leftrightarrow$	uu	kk	fi	ff	ii	tt	it
uu	tt	tt	ff	ff	tt	ff	ff
kk	tt	tt	ff	ff	tt	ff	ff
fi	ff	ff	tt	tt	ff	ff	ff
ff	ff	ff	tt	tt	ff	ff	ff
ii	tt	tt	ff	ff	tt	ff	ff
tt	ff	ff	ff	ff	ff	tt	tt
it	ff	ff	ff	ff	ff	tt	tt

Further, my weakest implication is not defined as $\neg A \vee B$. It is instead represented by the following table:

$\Rightarrow$	uu	kk	fi	ff	ii	tt	it
uu	tt	it	it	ii	it	tt	it
kk	it	tt	uu	ii	it	tt	it
fi	it	it	tt	it	it	tt	it
ff	tt	tt	tt	tt	tt	tt	tt
ii	it	it	it	uu	tt	tt	it
tt	uu	kk	fi	ff	ii	tt	it
it	it	uu	ii	fi	it	tt	tt

The property

$$A \wedge B \Rightarrow A \vee B$$

hold with the above implication. By inspecting the above table, one can see that other properties also hold. Examples are the following:
Identity:

$$A \Rightarrow A$$

contrapositive

$$A \Rightarrow B \Leftrightarrow \neg B \Rightarrow \neg A$$

using my negation whose table is the following:

	uu	kk	fi	ff	ii	tt	it
$\neg$	uu	kk	it	tt	ii	ff	fi

The above connective can be referred to as *symmetric negation*, at least with respect to the adopted interval $[-1.0, +1.0]$ and the subvariables, explained above.

Both De Morgan's properties hold:

$$\neg(A \vee B) \Leftrightarrow \neg A \wedge \neg B$$

$$\neg(A \wedge B) \Leftrightarrow \neg A \vee \neg B$$

Taking into consideration that **it** $\Leftrightarrow$ **tt** as a fuzzy approximation in this logic, other properties which hold are the following.

Implication commutativity:

$$(A \Rightarrow (B \Rightarrow C)) \Rightarrow (B \Rightarrow (A \Rightarrow C))$$

Implication prefixing:

$$(C \Rightarrow A) \Rightarrow ((B \Rightarrow C) \Rightarrow (B \Rightarrow A))$$

Implication sufixing:

$$(C \Rightarrow A) \Rightarrow ((A \Rightarrow B) \Rightarrow (C \Rightarrow B))$$

Implication distributivity:

$$(A \Rightarrow (B \Rightarrow C)) \Rightarrow ((A \Rightarrow B) \Rightarrow (A \Rightarrow C))$$

The law of excluded middle does not hold because the logic has more than two values.

IV. Conclusion

Having presented an uncertainty model and introduced the connectives of the present seven-valued logics, the latter can be easily implemented and used by researchers in computer science.

References

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